

Possible interpretation of the $Z_b(10610)$ and $Z_b(10650)$ in a chiral quark model

You-Chang Yang^{1,2}, Jialun Ping² ‡, Chengrong Deng³ and Hong-Shi Zong⁴

¹Department of Physics, Zunyi Normal College, Zunyi 563002, People's Republic of China

²Department of Physics, Nanjing Normal University, Nanjing 210097, People's Republic of China

³School of Mathematics and Physics, Chongqing Jiaotong University, Chongqing 400074, People's Republic of China

⁴Department of Physics, Nanjing University, Nanjing 210093, People's Republic of China

E-mail: jlping@njnu.edu.cn

Abstract. Motivated by the two charged bottomonium-like resonances $Z_b(10610)$ and $Z_b(10650)$ newly observed by the Belle collaboration, the possible molecular states composed of a pair of heavy mesons, $B\bar{B}$, $B\bar{B}^*$, $B^*\bar{B}^*$, $B_s\bar{B}$, etc (in S-wave), are investigated in the framework of chiral quark models by the Gaussian expansion method. The bound states $B\bar{B}^*$ and $B^*\bar{B}^*$ with quantum numbers $I(J^{PC}) = 1(1^{+-})$, which are good candidates for the $Z_b(10610)$ and $Z_b(10650)$ respectively, are obtained. Other three bound states $B\bar{B}^*$ with $I(J^{PC}) = 0(1^{++})$, $B^*\bar{B}^*$ with $I(J^{PC}) = 1(0^{++})$, $0(2^{++})$ are predicted. These states may be observed in open-bottom or hidden-bottom decay channel of highly excited Υ . When extending directly the quark model to the hidden color channel of the multi-quark system, more deeply bound states are found. Future experimental search of those states will cast doubt on the validity of applying the chiral constituent quark model to the hidden color channel directly.

PACS numbers: 12.39.Jh, 14.40.Lb, 14.40.Nd

Submitted to: *J. Phys. G*

1. Introduction

Very recently, the Belle collaboration observed two narrow peaks, which are named $Z_b(10610)$ and $Z_b(10650)$, in the $\pi^\pm \Upsilon(nS)$ ($n = 1, 2, 3$) and $\pi^\pm h_b(mP)$ ($m = 1, 2$) invariant mass spectra in the hidden-bottom decay channels of $\Upsilon(5S)$ [1]. The measured masses and widths of the two structures are,

$$M_{Z_b(10610)} = 10608.4 \pm 2.0 \text{ MeV}, \quad \Gamma = 15.6 \pm 2.5 \text{ MeV}$$

$$M_{Z_b(10650)} = 10653.2 \pm 1.5 \text{ MeV}, \quad \Gamma = 14.4 \pm 3.2 \text{ MeV}.$$

Analysis favors quantum numbers of $I^G(J^P) = 1^+(1^+)$ for both states. The $Z_b(10610)$ and $Z_b(10650)$ are both charged bottomonium-like resonances and the masses are very close to the thresholds of the open bottom channels $B^* \bar{B}(10604.6 \text{ MeV})$ and $B^* \bar{B}^*(10650.2 \text{ MeV})$, so the molecular states of S-wave $B^* \bar{B}$ and $B^* \bar{B}^*$ assignment are suggested by Belle collaboration.

In the hadron level, Nils A. Törnqvist investigated the deuteron-like meson-meson bound states by meson exchange model [2]. The study shows that the energy of isoscalars $B\bar{B}^*$ with $J^{PC} = 0^{-+}, 1^{++}$, $B^* \bar{B}^*$ with $J^{PC} = 0^{++}, 0^{-+}, 1^{+-}, 2^{++}$ are about 50 MeV below the corresponding $B\bar{B}^*$ and $B^* \bar{B}^*$ thresholds. No bound state, however, appears for isovectors. Recently, by taking the pseudoscalar, scalar and vector mesons exchange into account in the framework of the meson exchange model, Liu *et al.* found that the loosely bound states probably exists in S-wave $B\bar{B}^*$ [3, 4]. Very recently, Sun *et al.* believe that the $Z_b(10610)$, $Z_b(10650)$ are respectively $B^* \bar{B}$ and $B^* \bar{B}^*$ molecular state after considering S -wave and D -wave mixing [5, 6].

In the quark level, by solving the resonating group method equation, Liu *et al.* [7] also investigated the system composed of $[\bar{b}q][b\bar{q}]$, $[\bar{b}q]^*[b\bar{q}]$, $[\bar{b}q]^*[b\bar{q}]^*$ ($q = u, d, s$) by two chiral quark models in which the pseudoscalar, scalar and vector mesons exchange are taken. The isoscalars $B\bar{B}$, $B\bar{B}^*(C = +)$, $B^* \bar{B}^*(J = 2)$ favor molecular states. Bondar *et al.* also discussed the heavy quark spin structure of the $Z_b(10610)$ and $Z_b(10650)$ assuming that these are molecular state $B^* \bar{B}$ and $B^* \bar{B}^*$ [8]. By considering the contribution from the intermediate $Z_b(10610)$ and $Z_b(10650)$ states to the $\Upsilon(5S) \rightarrow \Upsilon(2S)\pi^+\pi^-$ decay process, the anomalous $\Upsilon(2S)\pi^+\pi^-$ production near the peak of $\Upsilon(5S)$ at $\sqrt{s} = 10.87 \text{ GeV}$ [9], observed by Belle collaboration, can be explained naturally [10]. The possibility of $Z_b(10610)$ and $Z_b(10650)$ being tetraquark states are discussed by the authors of Ref [11, 12]. The authors of Ref [13, 14]. think the tetraquark and molecular structure both can interpret the $Z_b(10610)$ and $Z_b(10650)$ in the QCD sum rule calculation. Further theoretical efforts concern the decay and mass of the $Z_b(10610)$ and $Z_b(10650)$ states discussed in Refs. [15, 16, 17].

Inspired by the new states $Z_b(10610)$ and $Z_b(10650)$ reported by Belle collaboration [1] and the related work, a systematical study of the possible S-wave $B\bar{B}$, $B\bar{B}^*$ and $B^* \bar{B}^*$ states is performed in this work. Here the B and \bar{B} stand for (B^+, B^0, B_s^0) and $(B^-, \bar{B}^0, \bar{B}_s^0)$ triplets, respectively. It is worthwhile to investigate the intrinsic structure of the $Z_b(10610)$, $Z_b(10650)$ and other possible exotic states with b, \bar{b}

quarks, especially in view of the great potential of finding new particles at Belle, BaBar, LHC and other collaborations.

To study the mass spectrum of above possible exotic states, two types of chiral quark models (ChQM) [18] are employed in this work. The numerical method, which is able to provide almost exact solutions, is very important in the study of few-body systems. Here, a high precision numerical method for few body system, which is different from the methods used in the previous work by other researchers, the Gaussian Expansion Method (GEM) is used. The detail of GEM can be found in Refs. [19, 20].

The paper is organized as follows. In the next section we introduce the Hamiltonian of the chiral quark models. Section 3 is devoted to discuss the wave function of possible molecular states $\mathcal{B}\bar{\mathcal{B}}$, $\mathcal{B}\bar{\mathcal{B}}^*$ and $\mathcal{B}^*\bar{\mathcal{B}}^*$. In Section 4, we present and analyze the results obtained in our calculation. Finally, the summary of the present work is given in the last section.

2. The chiral constituent quark model

In the ChQM, the Hamiltonian usually includes Goldstone-boson exchange in addition to color confinement and one-gluon-exchange (OGE). The chiral partner, σ -meson, is also usually introduced, although its existence is still in controversy [21]. The Hamiltonian of the ChQM used here is given as follows,

$$H = \sum_{i=1}^4 \left(m_i + \frac{\mathbf{p}_i^2}{2m_i} \right) - T_{CM} + \sum_{j>i=1}^4 (V_{ij}^C + V_{ij}^G + V_{ij}^\chi + V_{ij}^\sigma), \quad (1)$$

where $\chi = \pi, K, \eta$, T_{CM} is the kinetic energy operator of the center-of-mass motion of whole system.

The linear confining potential, which is suggested by lattice QCD calculation of $q\bar{q}$ system, can be written as

$$V_{ij}^C = \boldsymbol{\lambda}_i^c \cdot \boldsymbol{\lambda}_j^c (-a_c r_{ij} - \Delta). \quad (2)$$

For one-gluon-exchange, the potential takes the form

$$V_{ij}^G = \alpha_s \frac{\boldsymbol{\lambda}_i^c \cdot \boldsymbol{\lambda}_j^c}{4} \left[\frac{1}{r_{ij}} - \frac{2\pi}{3m_i m_j} (\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j) \delta(\mathbf{r}_{ij}) \right], \quad (3)$$

where, $\boldsymbol{\sigma}$, $\boldsymbol{\lambda}$ are the SU(2) Pauli matrices and the SU(3) Gell-Mann matrices, respectively. The $\boldsymbol{\lambda}$ should be replaced by $-\boldsymbol{\lambda}^*$ for the antiquark. In the non-relativistic quark model, the delta function $\delta(\mathbf{r}_{ij})$ should be regularized [22], because of the finite size of the constituent quark. The regulation is flavor dependent and reads [18, 23]

$$\delta(\mathbf{r}_{ij}) = \frac{1}{4\pi r_{ij} r_0^2(\mu)} e^{-r_{ij}/r_0(\mu)}, \quad (4)$$

where $r_0(\mu) = r_0/\mu$ and μ is the reduced mass of quark-quark (or antiquark) system. The wide energy covered from light to heavy quark requires an effective scale-dependent strong coupling constant α_s in Eq. (3) that cannot be obtained from the usual one-loop

expression of the running coupling constant because it diverges when $Q \rightarrow \Lambda_{QCD}$. Hence an effective scale-dependent strong coupling constant [18] is taken as

$$\alpha_s(\mu) = \frac{\alpha_0}{\ln \left[\frac{\mu^2 + \mu_0^2}{\Lambda_0^2} \right]}, \quad (5)$$

where μ_0 and Λ_0 are the free parameters.

For the mesons exchange, potential takes the form

$$V_{ij}^\pi = C(g_{ch}, \Lambda_\pi, m_\pi) \frac{m_\pi^2}{12m_i m_j} H_1(m_\pi, \Lambda_\pi, r_{ij}) (\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j) \sum_{a=1}^3 \lambda_i^a \cdot \lambda_j^a, \quad (6)$$

$$V_{ij}^K = C(g_{ch}, \Lambda_K, m_K) \frac{m_K^2}{12m_i m_j} H_1(m_K, \Lambda_K, r_{ij}) (\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j) \sum_{a=4}^7 \lambda_i^a \cdot \lambda_j^a, \quad (7)$$

$$V_{ij}^\eta = C(g_{ch}, \Lambda_\eta, m_\eta) \frac{m_\eta^2}{12m_i m_j} H_1(m_\eta, \Lambda_\eta, r_{ij}) (\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j) \times [\cos \theta_P (\lambda_i^8 \cdot \lambda_j^8) - \sin \theta_P (\lambda_i^0 \cdot \lambda_j^0)], \quad (8)$$

$$V_{ij}^\sigma = -C(g_{ch}, \Lambda_\sigma, m_\sigma) H_2(m_\sigma, \Lambda_\sigma, r_{ij}) \quad (9)$$

$$H_1(m, \Lambda, r) = \left[Y(mr) - \frac{\Lambda^3}{m^3} Y(\Lambda r) \right] \quad (10)$$

$$H_2(m, \Lambda, r) = \left[Y(mr) - \frac{\Lambda}{m} Y(\Lambda r) \right] \quad (11)$$

$$C(g_{ch}, \Lambda, m) = \frac{g_{ch}^2}{4\pi} \frac{\Lambda^2}{\Lambda^2 - m^2} m \quad (12)$$

where the σ exchange only occurs between the lightest quarks (u - or d -quark) which is different from Ref. [18] due to its non-strange nature. The strange scalar meson (with large mass) exchange is not taken into account in the present work because of its small effect. We adopt $\lambda^0 = \sqrt{\frac{2}{3}}I$ due to the normalization of SU(3) matrix. $Y(x)$ is the standard Yukawa function defined by $Y(x) = e^{-x}/x$ and the rest symbols have their usual meaning. The chiral coupling constant g_{ch} is determined from the πNN coupling constant through

$$\frac{g_{ch}^2}{4\pi} = \left(\frac{3}{5} \right)^2 \frac{g_{\pi NN}^2}{4\pi} \frac{m_{u,d}^2}{m_N^2}, \quad (13)$$

and flavor $SU(3)$ symmetry is assumed. The tensor term and the spin-orbital term have been omitted in the potentials since we consider only S-wave states.

The above model is denoted as ChQM1. To testing the effect of σ -exchange between the lightest and strange quark or strange quark pairs, and long-range color screening on the binding energy of the molecular states, the Salamanca version of the chiral quark model [18], which is referred as ChQM2, is also employed here. The screened confinement interaction in this model is

$$V_{ij}^C = \boldsymbol{\lambda}_i^c \cdot \boldsymbol{\lambda}_j^c \{ -a_c (1 - e^{-\mu_c r_{ij}}) + \Delta \}, \quad (14)$$

where μ_c is a color screening parameter. The other potentials are the same as the above with the exception that the σ -meson is exchanged between all the light quarks, u, d, s .

3. Wave function

The total wave function of multi-quark system can be written as,

$$\begin{aligned}\Psi_{JJ_z}^{I,I_z} &= |\xi\rangle |\eta\rangle^{II_z} \Phi_{JJ_z}, \\ \Phi_{JJ_z} &= [|\chi\rangle_S \otimes |\Phi\rangle_{L_T}]_{JJ_z}\end{aligned}\tag{15}$$

where $|\xi\rangle$, $|\eta\rangle^I$, $|\chi\rangle_S$, $|\Phi\rangle_{L_T}$ represent color singlet, isospin with I , spin with S and spacial wave function with angular momentum L_T , respectively.

All possible molecule structures composed of S-wave \mathcal{B} and $\bar{\mathcal{B}}$, which stand for (B^+, B^0, B_s^0) and $(B^-, \bar{B}^0, \bar{B}_s^0)$ triplets, respectively, are investigated in this work. According to the total isospin, the $P\bar{P}$ (pseudoscalar meson) and $V\bar{V}$ (vector meson) flavor wave functions are listed in Table 1. Another possible molecule structure for four-quark system is bottomonium+light meson. In this case, there is no interaction between colorless bottomonium and light meson (color dependent interaction is zero between two colorless cluster if no exchange term exists and there is also no Goldstone-boson exchange between heavy and light quarks), so no bound state can be formed in this case. Therefore, we do not take into account of this case in the present work.

Obviously the components $P\bar{V}$ and $V\bar{P}$ do not have definite C parity, one can get C parity $= \pm$ by $[P\bar{V} \pm \hat{C}(P\bar{V})]/\sqrt{2}$ and $[V\bar{P} \pm \hat{C}(V\bar{P})]/\sqrt{2}$ for the neutral states [2] such as $(\mathbf{I}, I_z) = (1, 0)$, $(0, 0)$ shown in Table 1 (All the orbital angular momenta are set to zero because we concentrate on ground states). Hence, the coefficient $C = \pm 1$ represent C -even and -odd parity respectively, which are different from that of Ref. [4, 3, 24], since we use normal convention of PDG [25] i.e. $B^0 = d\bar{b}$ and $\bar{B}^0 = \bar{d}b$. One can easy find that G parity $= \mp$ are corresponding to C parity $= \pm$ for these states with $(\mathbf{I}, I_z) = (1, 0)$. However, there is no interaction depending on C and G parity in the model Hamiltonian Eq.(1), so these two states with $\pm C$ and G parity must be degenerate in our calculation. The two states separated by comma in each row of Table 1 will be coupled in the calculation.

The spatial structures of molecular states are pictured in Fig. 1. The relative coordinates are defined as following,

$$\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2, \quad \mathbf{R} = \mathbf{r}_3 - \mathbf{r}_4, \tag{16}$$

$$\rho = \frac{m_1\mathbf{r}_1 + m_2\mathbf{r}_2}{m_1 + m_2} - \frac{m_3\mathbf{r}_3 + m_4\mathbf{r}_4}{m_3 + m_4}, \tag{17}$$

and the coordinate of the mass-center is

$$\mathbf{R}_{cm} = \sum_{i=1}^4 m_i \mathbf{r}_i / \sum_{i=1}^4 m_i, \tag{18}$$

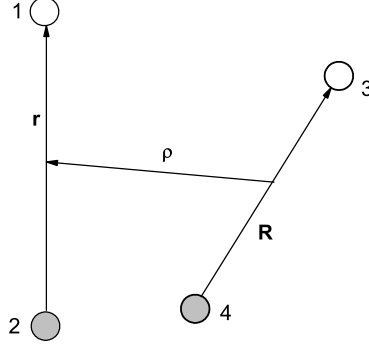
where m_i is the mass of the i th quark.

Then the outer product of space and spin wave functions is

$$\Phi_{JJ_z} = [[\phi_{lm}^G(\mathbf{r})\chi_{s_1 m_{s_1}}]_{J_1 M_1} [\psi_{LM}^G(\mathbf{R})\chi_{s_2 m_{s_2}}]_{J_2 M_2}]_{J_{12} M_{12}} \varphi_{\beta\gamma}^G(\rho)]_{JJ_z}. \tag{19}$$

Table 1. The flavor wave functions of the $\mathcal{B}\bar{\mathcal{B}}$, $\mathcal{B}\bar{\mathcal{B}}^*$ systems. “ C ” is the charge parity.

Isospin	$\mathcal{B}\bar{\mathcal{B}}$	$\mathcal{B}\bar{\mathcal{B}}^*$
$I=\frac{1}{2}$	$B^+\bar{B}_s^0$	$B^{*+}\bar{B}_s^0, B^+\bar{B}_s^{*0}$
	$B^0\bar{B}_s^0$	$B^{*0}\bar{B}_s^0, B^0\bar{B}_s^{*0}$
	$B_s^0\bar{B}^0$	$B_s^{*0}\bar{B}^0, B_s^0\bar{B}^{*0}$
	$B_s^0B^-$	$B_s^{*0}B^-, B_s^0B^{*-}$
$I=1$	$B^+\bar{B}^0$	$B^{*+}\bar{B}^0, B^+\bar{B}^{*0}$
	$\frac{1}{\sqrt{2}}(B^+B^- - B^0\bar{B}^0)$	$\frac{1}{2}[(B^{*+}B^- - B^{*0}\bar{B}^0) + C(B^{*-}B^+ - \bar{B}^{*0}B^0)],$ $\frac{1}{2}[(B^+B^{*-} - B^0\bar{B}^{*0}) + C(B^-B^{*+} - \bar{B}^0B^{*0})]$
	B^0B^-	$B^{*0}B^-, B^0B^{*-}$
$I=0(l)$	$\frac{1}{\sqrt{2}}(B^+B^- + B^0\bar{B}^0)$	$\frac{1}{2}[(B^{*+}B^- + B^{*0}\bar{B}^0) + C(B^{*-}B^+ + \bar{B}^{*0}B^0)],$ $\frac{1}{2}[(B^+B^{*-} + B^0\bar{B}^{*0}) + C(B^-B^{*+} + \bar{B}^0B^{*0})]$
$I=0(s)$	$B_s^0\bar{B}_s^0$	$\frac{1}{\sqrt{2}}(B_s^{*0}\bar{B}_s^0 + C\bar{B}_s^{*0}B_s^0), \frac{1}{\sqrt{2}}(B_s^0\bar{B}_s^{*0} + \bar{B}_s^0B_s^{*0})$

**Figure 1.** The relative coordinate for a meson and antimeson system. solid and hollow circles represent quarks and antiquarks, respectively.

In GEM, three relative motion wave functions are written as,

$$\phi_{lm}^G(\mathbf{r}) = \sum_{n=1}^{n_{max}} c_n N_{nl} r^l e^{-\nu_n r^2} Y_{lm}(\hat{\mathbf{r}}) \quad (20)$$

$$\psi_{LM}^G(\mathbf{R}) = \sum_{N=1}^{N_{max}} c_N N_{NL} R^L e^{-\zeta_N R^2} Y_{LM}(\hat{\mathbf{R}}) \quad (21)$$

$$\varphi_{\beta\gamma}^G(\rho) = \sum_{\alpha=1}^{\alpha_{max}} c_\alpha N_{\alpha\beta} \rho^\beta e^{-\omega_\alpha \rho^2} Y_{\beta\gamma}(\hat{\rho}) \quad (22)$$

Gaussian size parameters are taken as geometric progression

$$\nu_n = \frac{1}{r_n^2}, r_n = r_1 a^{n-1}, a = \left(\frac{r_{n_{max}}}{r_1} \right)^{\frac{1}{n_{max}-1}} \quad (23)$$

The expression of $\zeta_N, R_N, \omega_\alpha, \rho_\alpha$ in Eqs. (21) - (22) are similar to Eq. (23).

The physical state must be in color singlet, which can be constructed in two ways:

color-singlet and color octet,

$$|\xi_1\rangle = |\mathbf{1}_{12} \otimes \mathbf{1}_{34}\rangle, \quad |\xi_2\rangle = |\mathbf{8}_{12} \otimes \mathbf{8}_{34}\rangle. \quad (24)$$

The state in color octet channel is called **hidden color** states by analogy to states which appear in the nucleon-nucleon problem [26].

The total spin of $P\bar{P}$ and $P\bar{V}$ and $V\bar{P}$ system is only 0, 1 respectively. However, the $V\bar{V}$ system can coupling to total spin 0, 1 and 2.

4. Numerical results and discussion

Solving the Schrödinger equation

$$(H - E) \Psi_{J, J_z}^{I, I_z} = 0 \quad (25)$$

with Rayleigh-Ritz variational principle, the energies of normal mesons, $\mathcal{B}\bar{\mathcal{B}}$, $\mathcal{B}\bar{\mathcal{B}}^*$ and $\mathcal{B}^*\bar{\mathcal{B}}^*$ systems can be obtained by using different total wave functions, respectively.

To determine if the $\mathcal{B}\bar{\mathcal{B}}$, $\mathcal{B}\bar{\mathcal{B}}^*$ and $\mathcal{B}^*\bar{\mathcal{B}}^*$ systems are bound or not, the threshold of the system should be fixed. Clearly the threshold is governed by two corresponding meson masses. So one believes that a good fit of meson spectra, with the same parameters used in four-quark calculations, must be the most important criterium[23, 28, 27, 29, 30, 31]. Of course, there is another possible threshold for four-quark system, bottomonium+light meson, e.g., $\Upsilon(1S) + \rho$ for $IJ^P = 11^+$ channel. Generally the threshold in this case is lower, so the bound state $\mathcal{B}^{(*)}\bar{\mathcal{B}}^{(*)}$ will become a resonance in this channel. Since the transition from $\mathcal{B}^{(*)}\bar{\mathcal{B}}^{(*)}$ to bottomonium+light meson involves string rearrangement, we leave this for the future work.

In GEM, the calculated results of normal meson spectra (or the spectra of $\mathcal{B}\bar{\mathcal{B}}$, $\mathcal{B}\bar{\mathcal{B}}^*$ and $\mathcal{B}^*\bar{\mathcal{B}}^*$ systems) are converged with the number of gaussians $n_{max} = 7$ ($n_{max} = 7$, $N_{max} = 7$, $\alpha_{max} = 12$), and the size parameter r_n (r_n , R_N , ρ_α) running from 0.1 to 2 (2, 2, 6) fm. The convergence properties of the energies have been discussed in detail in Ref.[20]. The parameters and the normal meson spectra in two types of ChQM are listed in Table 2 and 3, respectively.

By solving the equation (25), the energy of the $\mathcal{B}\bar{\mathcal{B}}$, $\mathcal{B}\bar{\mathcal{B}}^*$ and $\mathcal{B}^*\bar{\mathcal{B}}^*$ systems can be obtained. If the binding energy, $\Delta E = M_{system} - M_{b\bar{q}} - M_{\bar{b}q}$ ($q = u, d, s$), is negative, then the system would be bound. According to the Table. 3, the thresholds of possible molecular states $\mathcal{B}\bar{\mathcal{B}}$, $\mathcal{B}\bar{\mathcal{B}}^*$, $\mathcal{B}^*\bar{\mathcal{B}}^*$ are easily listed in Table 4.

The color-singlet, color-octet channel, and channel coupling calculation of $\mathcal{B}\bar{\mathcal{B}}$, $\mathcal{B}\bar{\mathcal{B}}^*$, $\mathcal{B}^*\bar{\mathcal{B}}^*$ systems are done in the two types of ChQM, and the results are presented in Table 5-7, in which the results are denoted by “ $1 \otimes 1$ ”, “ $8 \otimes 8$ ” and “channel coupling”, respectively. Due to the strong interaction is invariant under the rotation of isospin, the different states corresponding to the different components of isospin I are degenerate, so we present the results for each total isospin I . From Table 5-7, we can see two models give very similar results.

In the S-wave $P\bar{P}$ system, the quantum numbers J^P are 0^+ . Apart from scalar meson σ , the pseudoscalar mesons e.g. π, K, η cannot be exchange in $\mathcal{B}\bar{\mathcal{B}}$ system because

Table 2. Parameters of two quark models. The masses of π, η in Eqs.(6)-(8) are got from the experimental data which are $m_\pi = 0.7 \text{ fm}^{-1}$, $m_\eta = 2.77 \text{ fm}^{-1}$, respectively; $m_\sigma, \Lambda_\pi, \Lambda_\eta, \theta_p$ are taken the same values as Ref.[18], namely $m_\sigma = 3.42 \text{ fm}^{-1}$, $\Lambda_\pi = \Lambda_\sigma = 4.2 \text{ fm}^{-1}$, $\Lambda_\eta = 5.2 \text{ fm}^{-1}$, $\theta_p = -15^\circ$, $g_{ch}^2/4\pi=0.54$

		ChQM1	ChQM2 [18]
Quark masses	$m_{u,d} \text{ (MeV)}$	313	313
	$m_s \text{ (MeV)}$	525	555
	$m_c \text{ (MeV)}$	1731	1752
	$m_b \text{ (MeV)}$	5100	5100
Confinement	$a_c \text{ (MeV fm}^{-1}\text{)}$	160	430
	$\Delta \text{ (MeV)}$	-131.1	181.1
	$\mu_c \text{ (fm}^{-1}\text{)}$	—	0.7
OGE	α_0	2.65	2.118
	$r_0 \text{ (MeV fm)}$	28.17	28.17
	$\mu_0 \text{ (MeV)}$	36.976	36.976
	$\Lambda_0 \text{ (fm)}$	0.075	0.113

Table 3. Numerical results of normal meson spectrum(unit: MeV) in the two quark models. The experimental data marked “Exp.” takes from the latest Particle Data Group[25], and the ground state of bottom $\eta_b(1s)$ is observed by BABAR Collaboration from the radiative transition $\Upsilon(3S) \rightarrow \gamma\eta_b$ [32]

Meson	ChQM1	ChQM2	Exp.	Meson	ChQM1	ChQM2	Exp.
π	140.1	153.2	139.57 ± 0.00035	D_s	1966.6	1991.8	1968.49 ± 0.34
K	496.2	484.9	493.677 ± 0.016	D_s^*	2091.1	2094.1	2112.3 ± 0.5
$\rho(770)$	775.3	773.1	775.49 ± 0.34	B^\pm	5284.7	5277.9	5279.15 ± 0.31
$K^*(892)$	917.9	907.7	896.00 ± 0.25	B^0	5284.7	5277.9	5279.53 ± 0.33
$\omega(782)$	703.7	696.6	782.65 ± 0.12	B^*	5324.3	5318.8	5325.1 ± 0.5
$\phi(1020)$	1016.8	1011.9	1019.422 ± 0.02	B_s^0	5360.6	5355.8	5366.3 ± 0.6
$\eta_c(1s)$	2995.7	2999.8	2980.3 ± 1.2	B_s^*	5403.6	5400.5	5412.8 ± 1.3
$J/\psi(1s)$	3097.6	3096.7	3096.916 ± 0.011	$\eta_b(1s)$	9384.6	9467.9	$9388.9^{+3.1}_{-2.3} \text{ (stat)}$
D^0	1882.2	1898.4	1864.84 ± 0.17	$\Upsilon(1s)$	9462.4	9504.7	9460.30 ± 0.26
D^*	2000.2	2017.3	2006.97 ± 0.19				

of the parity conservation. One can find in the Table 5 that the σ meson exchange do not contribute enough attraction to bind the $\mathcal{B}\bar{\mathcal{B}}$ system in color-singlet channel in the ChQM1. Due to the σ -exchange also occurs between ss pair and us or ds pairs, loosely bound states of $\mathcal{B}\bar{\mathcal{B}}$ with $I = 0(s)$, or $\frac{1}{2}$ are obtained in the ChQM2. Noteworthily, there is no one-gluon-exchange between two separate mesons just in this channel. More bound states are formed if we take the coupling of color-singlet and color-octet channels into account. Obviously, in the color octet channel, the color-magnetic terms of OGE between two separate colorful mesons contribute attraction to $\mathcal{B}\bar{\mathcal{B}}$ system, because of

Table 4. Threshold energy of $\mathcal{B}\bar{\mathcal{B}}, \mathcal{B}\bar{\mathcal{B}}^*, \mathcal{B}^*\bar{\mathcal{B}}^*$ in two quark models.(unit: MeV).

J	I	$M_1 M_2$	$E_{th}(\text{ChQM1})$	$E_{th}(\text{ChQM2})$
0	0,1	$\bar{B}^0 B^+$	10569.4	10555.8
	0	$B_s^- B_s^+$	10721.2	10711.6
	$\frac{1}{2}$	$B_s^- B^0 / B_s^+ B^-$	10645.3	10633.7
1	0,1	$\bar{B}^0 B^{*+}$	10609	10596.7
	0	$B_s^- B_s^{*+}$	10764.2	10756.3
	$\frac{1}{2}$	$B_s^- B^{*0} / B_s^+ B^{*-}$	10684.9	10674.6
	$\frac{1}{2}$	$B_s^{*-} B^0 / B_s^{*+} B^-$	10688.3	10678.4
0,1,2	0,1	$\bar{B}^{*0} B^{*+}$	10648.6	10637.6
	0	$B_s^{*-} B_s^{*+}$	10807.2	10801
	$\frac{1}{2}$	$B_s^{*-} B^{*0} / B_s^{*+} B^{*-}$	10727.9	10719.3

the requirement of total color-singlet of the state. In this case, due to the masses of u, d quarks are much smaller than the mass of b quark, the cross matrix between color-singlet and -octet channels of the color-magnetic interaction, which is in proportional to $1/(m_i m_j)$, is very large. So the energy of each $\mathcal{B}\bar{\mathcal{B}}$ system is depressed by it, which was discussed in detail in Ref. [23, 33].

Table 5. The binding energy (unit: MeV) of $\mathcal{B}\bar{\mathcal{B}}$. The "1 \otimes 1", "8 \otimes 8" and "channel coupling" represent $\mathcal{B}\bar{\mathcal{B}}$ in color-singlet, color-octet and coupling of color-singlet and color-octet channel, respectively.

Isospin	ChQM1			ChQM2		
	1 \otimes 1	8 \otimes 8	channel coupling	1 \otimes 1	8 \otimes 8	channel coupling
I= $\frac{1}{2}$	0.5	51.5	-17.5	-0.2	97.7	-2.6
I=1	0	0.1	-72.6	0	45.6	-29.9
I=0(l)	0	0.1	-72.6	0	45.6	-29.9
I=0(s)	0.3	77.4	0.2	-0.7	137.5	-1.5

For the $\mathcal{B}\bar{\mathcal{B}}^*$ system, the calculation results are listed in Table 6. The σ, π, η mesons can all be exchanged in such systems. Two bound states are both found in two quark models in color-singlet single channel. For $I = 1$ state, the binding energy are both about -1 MeV with regard to $\mathcal{B}\bar{\mathcal{B}}^*$ threshold in two quark models, and the distance of each pair quarks in ChQM1 are

$$\begin{aligned}
\sqrt{\langle r_{12}^2 \rangle} &= 0.63 \text{ fm}, \sqrt{\langle r_{34}^2 \rangle} = 0.63 \text{ fm}, \\
\sqrt{\langle r_{13}^2 \rangle} &= 2.1 \text{ fm}, \sqrt{\langle r_{24}^2 \rangle} = 2.1 \text{ fm}, \\
\sqrt{\langle r_{14}^2 \rangle} &= 2.02 \text{ fm}, \sqrt{\langle r_{23}^2 \rangle} = 2.18 \text{ fm}.
\end{aligned}$$

The distance between quark and quark or antiquark belonging to different mesons are larger than that between quark and antiquark belonging to the same meson, the results show a clear molecule structure. It is reasonable to interpret the state $Z_b(10610)$ reported

by Belle collaboration as the molecular state $B\bar{B}^*$ with $I(J^{PC}) = 1(1^{+-})$. For this state, there are also other thresholds, $\Upsilon(1S)\rho$ and $h_b\pi$ [15]. The energy of the state is a little higher than these thresholds. Because of the different color structures the states $B\bar{B}^*$ and $\Upsilon(1S)\rho$, $h_b\pi$ have, the transition involves the color structure rearrangement, $B\bar{B}^*$ may appear as a resonance in the $\Upsilon(1S)\rho$ and $h_b\pi$ channels [34]. The calculation of the transition, which is out of the scope of the present work, is left for future work. In our calculation, the $I = 0$ state without strange quark is also a bound state with binding energy -12.1 MeV in two quark models. In the ChQM2, in addition to the above two states, the $I = \frac{1}{2}$ and $0(s)$, also form a bound state for the σ -exchange contributes to these channels. The color-singlet singlet and hidden color channel coupling leads to that all the states are bound.

Table 6. The binding energy (unit: MeV) of $\mathcal{B}\bar{\mathcal{B}}^*$.

J^P	Isospin	ChQM1			ChQM2		
		$1 \otimes 1$	$8 \otimes 8$	channel coupling	$1 \otimes 1$	$8 \otimes 8$	channel coupling
1^+	$I=\frac{1}{2}$	0.4	-21.2	-89.8	-0.2	28.9	-40.9
	$I=1$	-1.3	-88.1	-164.3	-1.1	-35.5	-107.4
	$I=0(l)$	-12.1	-47.2	-122.6	-12.1	2.0	-69.8
	$I=0(s)$	0.35	14.8	-47.3	-1.2	77.5	-3.9

Table 7. The binding energy (unit: MeV) of $\mathcal{B}^*\bar{\mathcal{B}}^*$.

J^P	Isospin	ChQM1			ChQM2		
		$1 \otimes 1$	$8 \otimes 8$	channel coupling	$1 \otimes 1$	$8 \otimes 8$	channel coupling
0^+	$I=\frac{1}{2}$	0.4	-111.8	-172.9	-0.5	-56.6	-108.4
	$I=1$	-3.4	-194.4	-266.2	-3.0	-132.3	-195.5
	$I=0(l)$	0.4	-104.9	-175.9	0.4	-49.8	-115.3
	$I=0(s)$	0.3	-58.9	-112.2	0	7.6	-37.7
1^+	$I=\frac{1}{2}$	0.4	-71.4	-129.0	-0.3	-22.5	-73.9
	$I=1$	-1.2	-135.7	-201.5	-0.9	-83.2	-144.6
	$I=0(l)$	0.4	-94	-160.1	0.5	-44.9	-107.6
	$I=0(s)$	0.3	-34.9	-86.0	-0.2	26.5	-21.2
2^+	$I=\frac{1}{2}$	0.4	-2.6	-56.4	0	36.1	-19.3
	$I=1$	0.3	-37.6	-97.0	0.4	-1.9	-62.6
	$I=0(l)$	-11.1	-74.3	-133.0	-11.0	-35.7	-95.1
	$I=0(s)$	0.3	8.4	-41.0	-1.1	61.1	-5.7

The S-wave $\mathcal{B}^*\bar{\mathcal{B}}^*$ systems have quantum numbers $J^{PC} = 0^{++}$, 1^{+-} , and 2^{++} for the neutral states. The π , η , σ mesons can all be exchanged for $I=0(l)$ and $I=1$, while only η is exchangeable for $I=0(s)$ and $\frac{1}{2}$ states. The σ interaction is always attractive between the lightest u , d quarks. According to Eq. (6), the π -exchange is attractive for the states with $I(J^P) = 1(0^+)$, $1(1^+)$, $0(2^+)$ and makes these states are all bound

states, which are shown in Table 7 in the color-singlet single channel calculation. The $\mathcal{B}^*\bar{\mathcal{B}}^*$ with $I(J^P) = 1(1^+)$ has binding energy about -1 MeV in two quark models, and the distance of each pair quarks are

$$\begin{aligned}\sqrt{\langle r_{12}^2 \rangle} &= 0.64 \text{ fm}, \sqrt{\langle r_{34}^2 \rangle} = 0.64 \text{ fm}, \\ \sqrt{\langle r_{13}^2 \rangle} &= 2.2 \text{ fm}, \sqrt{\langle r_{24}^2 \rangle} = 2.2 \text{ fm}, \\ \sqrt{\langle r_{14}^2 \rangle} &= 2.11 \text{ fm}, \sqrt{\langle r_{23}^2 \rangle} = 2.27 \text{ fm}.\end{aligned}$$

The assignment of the newly observed state $Z_b(10650)$ to a molecular state $\mathcal{B}^*\bar{\mathcal{B}}^*$ with $I(J^{PC}) = 1(1^{+-})$ is favored. In the hidden color channel, almost all the states, except the one with $IJ^P = 02^+$ and hidden strange, are become bound. Again the channel coupling between color-singlet and hidden color channels makes all the states bound. In the present work, the S - D mixing of Z_b is not taken into account. the mixing will be important for states with energy on the threshold. From the calculation of deuteron, we estimate the S - D mixing will increase the binding energy of Z_b about 2 MeV.

5. Summary

In the framework of chiral quark model, a systematical study of the mass spectra of $\mathcal{B}\bar{\mathcal{B}}$, $\mathcal{B}\bar{\mathcal{B}}^*$ and $\mathcal{B}^*\bar{\mathcal{B}}^*$ systems is performed. The states $\mathcal{B}\bar{\mathcal{B}}^*$ and $\mathcal{B}^*\bar{\mathcal{B}}^*$ with quantum numbers $I(J^{PC}) = 1(1^{+-})$ are shown to be bound, which are respectively good candidates for the charged bottomonium-like resonances $Z_b(10610)$ and $Z_b(10650)$ newly observed by Belle collaboration. The color-singlet single channel calculation also shows that the states $B\bar{B}^*$ with $I(J^{PC}) = 0(1^{++})$, $B^*\bar{B}^*$ with $I(J^{PC}) = 1(0^{++})$, $0(2^{++})$ are bound states with a few MeV binding energy.

Recently Belle collaboration reported their high precision measurement of bottomonium mass: $M[\Upsilon(5S)] = 10.87 \text{ GeV}$ [9]. If the molecular states $B^*\bar{B}^*$ with $I(J^P) = 1(0^+)$ really exist, it could be observed in final state $\Upsilon(1S)\rho$ at the Belle, BaBar, LHC and other collaborations. Due to the phase space limitation, the isoscalar states $B\bar{B}^*(J^P = 1^+)$ and $B^*\bar{B}^*(J^P = 2^+)$ may be observed in decays of excited bottomonium which above the $\Upsilon(5S)$.

The σ -exchange plays important role for binding the $\mathcal{B}\bar{\mathcal{B}}$, $\mathcal{B}\bar{\mathcal{B}}^*$ and $\mathcal{B}^*\bar{\mathcal{B}}^*$ with $I = \frac{1}{2}$ and $0(s)$ in the ChQM2. To search these molecular states in the future experiment will test the contribution of the σ -exchange in the chiral constituent quark model.

The hidden color channel effect is complicated in multi-quark systems. Here we extend directly the quark model for colorless cluster to the colorful cluster in the study of $\mathcal{B}\bar{\mathcal{B}}$, $\mathcal{B}\bar{\mathcal{B}}^*$ and $\mathcal{B}^*\bar{\mathcal{B}}^*$ systems. In our calculation, the color-octet channel plays a dominate role in producing deeply bound states. If the quark-antiquark interaction in color singlet can be extended directly to color-octet by Casimir scaling [35], then the OGE interaction will be attractive between two color-octet cluster in some multi-quark systems. So it is inevitably to produce deeply bound states for hidden-bottom states because of the too small kinetic energy. More experimental data on bottomonium-like

resonances are needed to check the Casimir scaling, and cast doubt on the validity of applying the chiral constituent quark model to the hidden color channel directly.

Acknowledgments

This work is supported partly by the National Science Foundation of China under Contract Nos. 11047023, 11035006, 11175088, 11047140, and the Science Foundation of Guizhou Provincial Education Department under Grant No. 20100084, and the Science Foundation of Guizhou Science and Technology Department under Grant No. J[2011]2364, and the key support discipline of Guizhou province No. [2011]275.

References

- [1] Adachi I, Adamczyk K *et al.* (Belle Collaboration) arXiv:1105.4583v3[hep-ex]; Bondar A, Garmashar A *et al.* 2012 *Phys. Rev. Lett.* **108** 122001, arXiv:1110.2251 [hep-ex]
- [2] Tornqvist N A 1994 *Z. Phys. C* **61** 525
- [3] Liu Y R, Liu X, Deng W Z and Zhu S L 2008 *Eur. Phys. J. C* **56** 63
- [4] Liu X, Luo Z G, Liu Y R and Zhu S L 2009 *Eur. Phys. J. C* **61** 411
- [5] Sun Zhi-Feng, He Jun, Liu Xiang, *et al.* 2012 *Phys. Rev. D* **84** 054002
- [6] Ohkoda S, Yamaguchi Y, Yasui S, *et al.* arXiv:1111.2921 [hep-ph]
- [7] Liu Y R and Zhang Z Y 2009 *Phys. Rev. C* **80** 015208
- [8] Bondar A E, Garmash A, Milstein A I *et al.* arXiv:1105.4473 [hep-ph]
- [9] Chen K F *et al.* (Belle Collaboration) 2008 *Phys. Rev. Lett.* **100** 112001
- [10] Chen D Y, Liu X and Zhu S L 2011 *Phys. Rev. D* **84** 074016
- [11] Ali Ahmed, Hambrock Christian and Wang Wei 2012 *Phys. Rev. D* **85** 054011
- [12] Guo T, Cao L, Zhou M Z, and Chen H, arXiv:1106.2284
- [13] Zhang J R, Zhong M and Huang M Q 2011 *Phys. Lett. B* **704**, 312
- [14] Cui Chun-Yu, Liu Yong-Lu and Huang Ming-Qiu 2012 *Phys. Rev. D* **85** 074014
- [15] Cleven Martin and Guo Feng-Kun and Hanhart Christoph, *et al.* 2012 *Eur. Phys. J. A* **47** 120
- [16] Yubing Dong, Amand Faessler, Thomas Gutsche *et al.* 2012 arXiv:1203.1894 [hep-ph]
- [17] Li M T, Wang W L, Dong Y B, *et al.* 2012 arXiv:1204.3959[hep-ph]
- [18] Vijande J, Fernandez F and Valcarce A 2005 *J. Phys. G* **31** 481
- [19] Hiyama E, Kino Y and Kamimura M 2003 *Prog. Part. Nucl. Phys.* **51** 223
- [20] Yang Y C, Deng C R, Ping J L and Goldman T 2009 *Phys. Rev. D* **80** 114023
- [21] Kaiser N, Grestendorfer S and Weise W 1998 *Nucl. Phys. A* **637** 395; Oset E, Toki H, Mizobe M and Takahashi T T 2000 *Prog. Theor. Phys.* **103** 351; Kaskulov M M and Clement H 2004 *Phys. Rev. C* **70** 014002; Chen L Z, Pang H R, Huang H X, Ping J L and Wang F 2007 *Phys. Rev. C* **76** 014001
- [22] Bhaduri R K, Cohler L E and Nogami Y 1980 *Phys. Rev. Lett.* **44** 1369
- [23] Weinstein J D and Isgur N 1982 *Phys. Rev. Lett.* **48** 659; 1983 *Phys. Rev. D* **27** 588; 1990 *Phys. Rev. D* **41** 2236
- [24] Liu X, Liu Y R, Deng W Z and Zhu S L 2008 *Phys. Rev. D* **77** 034003; arXiv:0711.0494 [hep-ph]
- [25] Nakamura K *et al.* (Particle Data Group), 2010 *J. Phys. G* **37** 075021 and 2011 partial update for the 2012 edition
- [26] Huang H X, Xu P, Ping J L and Wang F 2011 *Phys. Rev. C* **84** 064001
- [27] Manohar A V, Wise M B 1993 *Nucl. Phys. B* **399** 17
- [28] Silvestre-Brac B and Gignoux C 1985 *Phys. Rev. D* **32** 743
- [29] Silvestre-Brac B and Semay C 1993 *Z. Phys. C* **57** 273; **59** 457
- [30] Brink D M and Stancu F 1998 *Phys. Rev. D* **57** 6778

- [31] Janc D and Rosina M 2004 *Few-Body Systems* **35** 175
- [32] Aubert B *et al.* [BABAR Collaboration] 2008 *Phys. Rev. Lett.* **101** 071801; Grenieret P hep-ex:0809.1672.
- [33] Yang Y C and Ping J L 2010 *Phys. Rev. D* **81** 114025
- [34] Ping J L, Deng C R, Wang F and Goldman T 2008 *Phys. Lett. B* **659** 607
- [35] Bali G S 2000 *Phys. Rev. D* **62** 114503